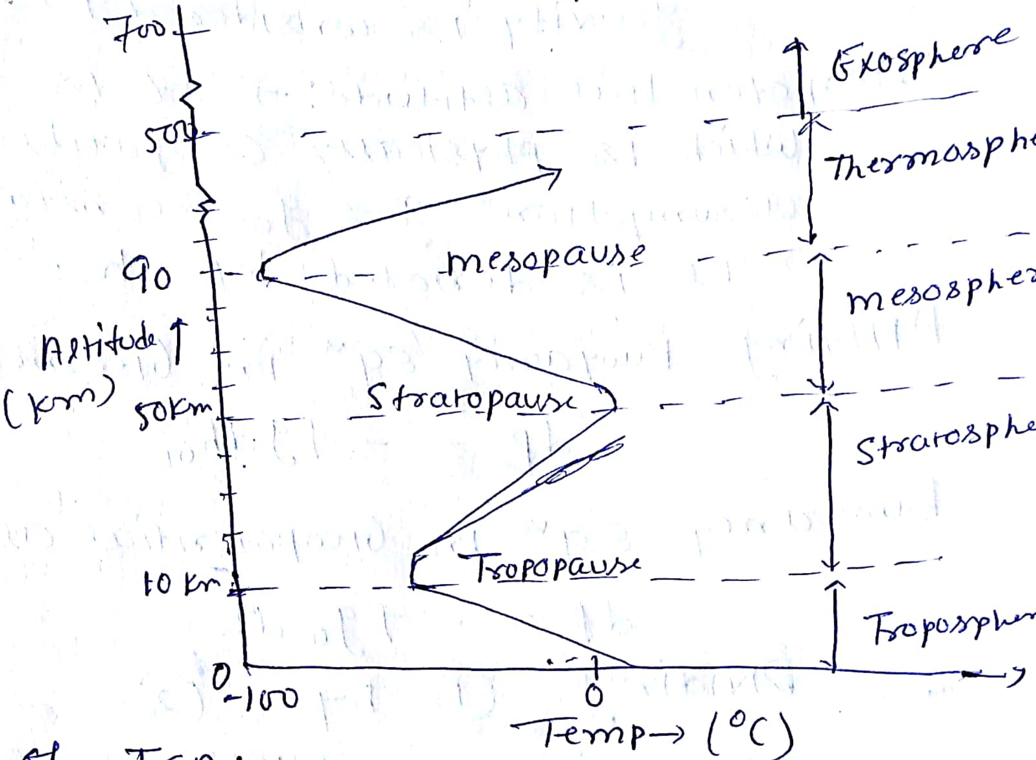
(2) Ans: → The earth's atmosphere is a gaseous blanket around the earth which is divided into the five regions based on certain intrinsic features. These five regions are: (I) Troposphere, (II) Stratosphere, (III) mesosphere, (IV) Ionosphere and (V) Exosphere.



Importance of ISA:—

→ The properties of earth's atmosphere like pressure, temperature and density vary not only with height above the earth's surface but also with the location on earth, from day to day and even during the day.

→ The performance of an airplane is dependent on the physical properties of the earth's atmosphere. Hence for the purpose of comparing

(a) the performance of different airplanes and

(b) the performance of the same airplane measured in flight tests on different days, a set of values for atmospheric properties have been agreed upon, which represent average conditions prevailing for most of the year, in Europe and North America.

→ This set of values called as international standard atmospher (ISA) is prescribed by ICAO (International Civil aviation organization). It is defined by pressure and temperature at mean sea level, and variation of temperature with altitude up to 32 km.

→ from these prescribed value it is possible to find other required physical characteristics (pressure, temperature and density, etc.) at any chosen altitude.

⇒ features of ISA: →

The main features of ISA are the standard sea level values and the variation of temperature with altitude. The air is assumed as dry perfect gas.

The standard sea level conditions are as follow

$$\text{Temperature } (T_0) = 288.15\ K = 15^\circ C$$

$$\text{Pressure } (P_0) = 101325\ N/m^2$$

Rate of change of temperature:

$= -6.5$ K/km upto 11 km

$= 0$ K/km from 11 to 20 km

$= 1$ K/km from 20 to 32 km

(3) Ans: → Given

$P_0 = 101325$ pa , $T_0 = 288.15$ K , $R_{air} = 287$ J/kgK

$\rho_0 = 1.2256$ kg/m^3

At 11 km altitude

$$T_{11} = T_0 - Lh$$
$$= 288.15 - 0.0065 \times 11 \times 1000$$
$$= 216.65 \text{ K}$$

$$\Rightarrow \frac{P_{11}}{P_0} = \left(\frac{T_{11}}{T_0}\right)^{g/LR}$$

$$\Rightarrow P_{11} = 101325 \times \left(\frac{216.65}{288.15}\right)^{5.2586} = 22614.489 \text{ pa}$$

$$\Rightarrow \frac{\rho_{11}}{\rho_0} = \left(\frac{T_{11}}{T_0}\right)^{g/LR - 1}$$

~~ρ_{11} = 288.15 ×~~

$$\rho_{11} = 1.2256 \times \left(\frac{216.65}{288.15}\right)^{4.2586} = 0.3638 \text{ kg/m}^3$$

At 15 km altitude

$$\Rightarrow \frac{P_{15}}{P_{11}} = \frac{\rho_{15}}{\rho_{11}} = e^{-\frac{g}{RT_{11}}(h_2 - h_1)}$$
$$= e^{-\frac{9.807}{287 \times 216.65} \times (15-11) \times 1000}$$
$$= 0.5321$$

$$\Rightarrow P_{15} = 0.5321 \times 22614.489 = 12033.1696 \text{ pa}$$ Ans

$$\Rightarrow \rho_{15} = 0.5321 \times 0.3638 = 0.1936 \text{ kg/m3}$$ Ans

$$\Rightarrow T_{15} = T_{11} = 216.65 \text{ K}$$ Ans:

$$\Rightarrow \mu_{15} = 1.458 \times 10^{-6} \left[\frac{T_{15}^{3/2}}{T_{15} + 110.4}\right]$$

$$= 1.458 \times 10^{-6} \left[\frac{(216.65)^{3/2}}{216.65 + 110.4}\right] = 14.216 \times 10^{-6} \text{ kg/m-s}$$

(4) Ans: →

$$dp = -\rho g\, dh \quad \text{--- (1)} \quad (\text{Buoyancy Eq}^n)$$

$$\frac{P}{\rho} = RT \quad \text{--- (2)} \quad (\text{Ideal gas law})$$

from (2) $\rho = \frac{P}{RT}$, putting it in (1)

$$dp = -\frac{Pg}{RT} dh$$

$$\frac{dP}{P} = -\frac{g}{R}\frac{dh}{T} \quad \text{--- (3)}$$

In troposphere, Temperature reduces linearly with a lapse rate of 'L' K/Km

so,

$$\left.\begin{aligned} T &= T_0 - Lh \\ T_1 &= T_0 - Lh_1 \\ T_2 &= T_0 - Lh_2 \end{aligned}\right\} \leftarrow (A)$$

from (3) and (A)

$$\int_{P_1}^{P_2} \frac{dP}{P} = -\frac{g}{R}\int_{h_1}^{h_2} \frac{dh}{T_0 - Lh}$$

$$\ln\frac{P_2}{P_1} = \frac{g}{LR}\ln\left(\frac{T_2}{T_1}\right)$$

$$\boxed{\left(\frac{P_2}{P_1}\right) = \left(\frac{T_2}{T_1}\right)^{g/LR}} \quad \text{--- (4)}$$

from Eq^n (2), we can write

$$\rho = \frac{P}{RT}$$

$$\frac{\rho_2}{\rho_1} = \frac{P_2}{\cancel{R}T_2} \times \frac{\cancel{R}T_1}{P_1}$$

$$= \frac{P_2}{P_1} \times \frac{T_1}{T_2} = \frac{P_2}{P_1} \times \left(\frac{T_2}{T_1}\right)^{-1}$$

from (4)

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{g/LR} \times \left(\frac{T_2}{T_1}\right)^{-1}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{g/LR - 1}}$$

(5) Ans :- → Given $\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$

(a) Displacement thickness $(\delta^*) = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$

$$\delta^* = \int_0^\delta \left(1 - \frac{y^{1/7}}{\delta^{1/7}}\right) dy$$

$$= \left[y - \frac{7}{8}\frac{y^{8/7}}{\delta^{1/7}}\right]_0^\delta$$

$$= \left[\delta - \frac{7}{8}\delta\right]$$

$$= \frac{1}{8}\delta$$

$$\delta^* = \frac{1}{8} \times 25 = 3.125 \text{ mm}$$ Ans

(b) momentum thickness, $(\theta) = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$

$$\Rightarrow \theta = \int_0^\delta \left(\frac{y}{\delta}\right)^{1/7}\left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy$$

$$= \int_0^\delta \left[\left(\frac{y}{\delta}\right)^{1/7} - \left(\frac{y}{\delta}\right)^{2/7}\right] dy$$

$$= \left[\frac{7}{8}\frac{y^{8/7}}{\delta^{1/7}} - \frac{7}{9}\frac{y^{9/7}}{\delta^{2/7}}\right]_0^\delta$$

$$\theta = \left[\frac{7}{8}\delta - \frac{7}{9}\delta\right]$$

$$= \frac{7}{72}\delta = \frac{7}{72} \times 25$$

$$\boxed{\theta = 2.4306\ mm}$$ Ans

(c) Energy thickness $(\delta^{**}) = \int_0^{\delta} \frac{u}{U_\infty}\left(1 - \left(\frac{u}{U_\infty}\right)^2\right) dy$

$$\delta^{**} = \int_0^{\delta} \left(\frac{y}{\delta}\right)^{1/7}\left[1 - \left(\frac{y}{\delta}\right)^{2/7}\right] dy$$

$$= \int_0^{\delta} \left[\frac{y^{1/7}}{\delta^{1/7}} - \frac{y^{3/7}}{\delta^{3/7}}\right] dy$$

$$= \left[\frac{7}{8} \cdot \frac{y^{8/7}}{\delta^{1/7}} - \frac{7}{10}\frac{y^{10/7}}{\delta^{3/7}}\right]$$

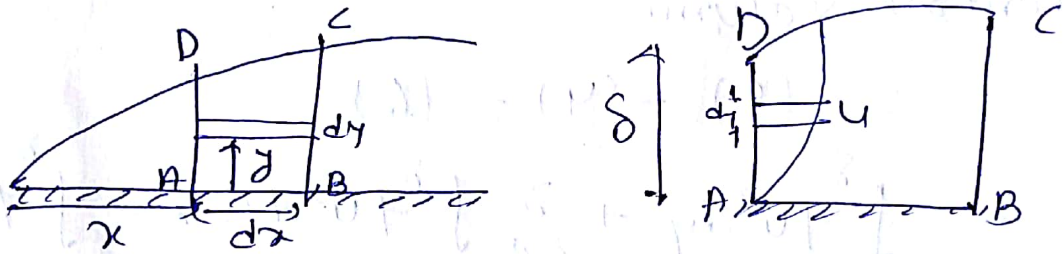
$$= \frac{7}{8}\delta - \frac{7}{10}\delta = \frac{7}{40}\delta$$

$$\delta^{**} = \frac{7}{40} \times 25 = 4.375\ mm$$ Ans

(6) Ans: →

Von karman suggested a method based on the momentum equation by the use of which wall shear stress and drag force on a flat plate can be determined, if the velocity distribution of boundary layer is known.

Fig. below shows a fluid flowing over a flat plate with speed U_∞. Consider a small length 'dx' of the plate at a distance 'x' from the L.E. of the plate; ~~and~~ Assume the width of plate is unity.



mass flow rate of fluid entering through 'AD'

$$\dot{m}_1 = \int_0^\delta \rho u \, dy \quad \text{(1)}$$

mass flow rate of fluid leaving through 'BC'

$$\dot{m}_2 = \int_0^\delta \rho u \, dy + \frac{\partial}{\partial x} \int_0^\delta \rho u \, dy \, dx \quad \text{(2)}$$

mass flow rate of fluid entering through 'DC', $\dot{m}_3 = \dot{m}_2 - \dot{m}_1$

$$= \cancel{\int_0^\delta \rho u \, dy} + \frac{\partial}{\partial x} \int_0^\delta \rho u \, dy \, dx - \cancel{\int_0^\delta \rho u \, dy}$$

$$\dot{m}_3 = \frac{\partial}{\partial x} \int_0^\delta \rho u \, dy \, dx \quad \text{(3)}$$

Rate of momentum of fluid entering through face 'AD'

$$= \int_0^\delta \rho u^2 \, dy \quad \text{(4)}$$

Rate of momentum of fluid leaving the face 'BC'

$$= \int_0^\delta \rho u^2 \, dy + \frac{\partial}{\partial x} \int_0^\delta \rho u^2 \, dy \, dx \quad \text{(5)}$$

Rate of momentum of fluid entering through face 'CD'

$$= \frac{\partial}{\partial x}\int_0^\delta \rho u u_\infty \, dx\, dy \quad \text{---} \quad (6)$$

Net rate of change of momentum of fluid in control volume

$$= (5) - (4) - (6)$$

$$= \cancel{\int_0^\delta \rho u^2 dy} + \frac{\partial}{\partial x}\int_0^\delta \rho u^2 dy\, dx - \cancel{\int_0^\delta \rho u^2 dy}$$

$$- \frac{\partial}{\partial x}\int_0^\delta \rho u u_\infty \, dx\, dy$$

$$= -\rho u_\infty^2 \frac{\partial}{\partial x}\int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy\, dx$$

$$= -\rho u_\infty^2 \frac{\partial \theta}{\partial x} dx \quad \text{---} \quad (A)$$

Drag force acts on plate

$$F_D = -\tau_0 \times dx \times 1 \quad \text{---} \quad (B)$$

Equating (A) and (B)

$$\cancel{-}\tau_0 \times \cancel{dx} \times 1 = \cancel{-}\rho u_\infty^2 \frac{\partial \theta}{\partial x} \cancel{dx}$$

$$\boxed{\frac{\tau_0}{\rho u_\infty^2} = \frac{\partial \theta}{\partial x}} \quad \text{---} \quad (7)$$

Equation (7) is known as Von Karman momentum equation which is valid for both laminar and turbulent flow.

(7) Ans: →

Geometric altitude (h_G): – The altitude measured ~~from~~ above the earth surface is known as geometric altitude.

→ Variation of ~~gravity~~ acceleration due to gravity is considered in this altitude.

Geopotential altitude: → It is fictitous altitude which is physically compatible with the assumption $g = g_0$ = constant.

→ it is denoted by h.

Applying Buoyancy eqn in Geometric altitude

$$dp = -\rho g \, dh_G \quad \text{——} (1)$$

Buoyancy eqn in Geopotential altitude

$$dp = -\rho g_0 \, dh \quad \text{——} (2)$$

Dividing (1) by (2)

$$\frac{\cancel{dp}}{\cancel{dp}} = \frac{\cancel{-}\cancel{\rho} g \, dh_G}{\cancel{-}\cancel{\rho} g_0 \, dh}$$

$$1 = \frac{g}{g_0} \frac{dh_G}{dh} \quad \text{——} (3)$$

Relation between 'g' and 'g_0'

$$g = g_0 \left(\frac{R}{R+h_G}\right)^2 \quad \text{——} (4)$$

from (3) and (4)

$$1 = \left(\frac{R}{R+h_G}\right)^2 \frac{dh_G}{dh}$$

$$\int_0^h dh = R^2 \int_0^{h_G} \frac{dh_G}{(R+h_G)^2}$$

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$$h = R^2 \times \frac{1}{-1} \left[\frac{1}{R+h_G} \right]_0^{h_G}$$

$$h = -R^2 \left[\frac{1}{R+h_G} - \frac{1}{R} \right]$$

$$= -R^{\cancel{2}} \left[\frac{\cancel{R} - \cancel{R} - h_G}{\cancel{R}(R+h_G)} \right]$$

$$\boxed{h = \left(\frac{R}{R+h_G} \right) h_G}$$